Isospin mixing in the nucleon and ^{4}He and the nucleon strange electric form factor

M. Viviani[1], R. Schiavilla[2,3], B. Kubis[4], R. Lewis[5],
L. Girlanda[1], A. Kievsky[1], L.E. Marcucci[1], and S. Rosati[1]
[1]*INFN, Sezione di Pisa, and Department of Physics, University of Pisa, I-56127 Pisa, Italy*
[2]*Jefferson Lab, Newport News, VA 23606, USA*
[3]*Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[4]*HISKP (Theorie), Universität Bonn, Nussallee 14–16, D-53115 Bonn, Germany*
[5]*Department of Physics, University of Regina, Regina, Saskatchewan, Canada, S4S 0A2*

In order to isolate the contribution of the nucleon strange electric form factor to the parity-violating asymmetry measured in $^4\mathrm{He}(\vec{e},e')^4\mathrm{He}$ experiments, it is crucial to have a reliable estimate of the magnitude of isospin-symmetry-breaking (ISB) corrections in both the nucleon and ^{4}He. We examine this issue in the present letter. Isospin admixtures in the nucleon are determined in chiral perturbation theory, while those in ^{4}He are derived from nuclear interactions, including explicit ISB terms. A careful analysis of the model dependence in the resulting predictions for the nucleon and nuclear ISB contributions to the asymmetry is carried out. We conclude that, at the low momentum transfers of interest in recent measurements reported by the HAPPEX collaboration at Jefferson Lab, these contributions are of comparable magnitude to those associated with strangeness components in the nucleon electric form factor.

PACS numbers: 14.20.Dh,25.30.Bf,12.15.Ji

One of the challenges of modern hadronic physics is to determine, at a quantitative level, the role that quark-antiquark pairs, and in particular $s\bar{s}$ pairs, play in the structure of the nucleon. Parity-violating (PV) electron scattering from nucleons and nuclei offers the opportunity to investigate this issue experimentally. The PV asymmetry (A_{PV}) arises from interference between the amplitudes due to exchange of photons and Z-bosons, which couple respectively to the electromagnetic (EM) and weak neutral (NC) currents. These currents involve different combinations of quark flavors, and therefore measurements of A_{PV}, in combination with electromagnetic form factor data for the nucleon, allow one to isolate, in principle, the electric and magnetic form factors G_E^s and G_M^s, associated with the strange-quark content of the nucleon.

Experimental determinations of these form factors have been reported recently by the Jefferson Lab HAPPEX [1] and G0 [2] Collaborations, Mainz A4 Collaboration [3], and MIT-Bates SAMPLE Collaboration [4]. These experiments have scattered polarized electrons from either unpolarized protons at forward angles [1, 2, 3] or unpolarized protons and deuterons at backward angles [4]. The resulting PV asymmetries are sensitive to different linear combinations of G_E^s and G_M^s as well as the nucleon axial-vector form factor G_A^Z. However, no robust evidence has emerged so far for the presence of strange-quark effects in the nucleon.

Last year, the HAPPEX Collaboration [5, 6] at Jefferson Lab reported on measurements of the PV asymmetry in elastic electron scattering from ^{4}He at four-momentum transfers of 0.091 (GeV/c)^2 and 0.077 (GeV/c)^2. Because of the $J^\pi{=}0^+$ spin-parity assignments of this nucleus, transitions induced by magnetic and axial-vector currents are forbidden, and therefore these measurements can lead to a direct determination of the strangeness electric form factor G_E^s [7, 8], provided that isospin symmetry breaking (ISB) effects in both the nucleon and ^{4}He, and relativistic and meson-exchange (collectively denoted with MEC) contributions to the nuclear EM and weak vector charge operators, are negligible. A realistic calculation of these latter contributions [8] found that they are in fact tiny at low momentum transfers. The goal of the present letter is to provide a quantitative estimate of ISB corrections to the PV asymmetry.

In the following analysis, we only need to consider the time components of the EM current and vector part of the weak NC current—the weak vector charge referred to above [8]. We account for isospin symmetry breaking in both the nucleon and α-particle. We first discuss it in the nucleon.

Ignoring radiative corrections, the EM and weak vector charge operators can be decomposed as

$$j^{\mu=0}_{\rm EM} \;=\; j^{(0)} + j^{(1)} \ , \tag{1}$$

$$j^{\mu=0}_{\rm NC} \;=\; -4s_W^2 j^{(0)} + (2-4s_W^2) j^{(1)} - j^{(s)} \ , \tag{2}$$

where $j^{(0)}$ and $j^{(1)}$ are respectively the isoscalar and isovector components of the EM charge operators, $j^{(s)}$ is the (isoscalar) component due to strange-quark contributions, and $s_W^2=\sin^2\theta_W$ contains the Weinberg mixing angle. In a notation similar to that adopted by the authors of Ref. [9], we introduce form factors corresponding to the following matrix elements of $j^{(0)}$ and $j^{(1)}$ between proton (p) and neutron (n) states:

$$\langle p|j^{(0)}|p\rangle \;\to\; G_E^0(Q^2) + G_E^{\not{0}}(Q^2) \ , \tag{3}$$

$$\langle n|j^{(0)}|n\rangle \;\to\; G_E^0(Q^2) - G_E^{\not{0}}(Q^2) \ , \tag{4}$$

$$\langle p|j^{(1)}|p\rangle \;\rightarrow\; G_E^{1}(Q^2)+G_E^{\not{1}}(Q^2)\ , \qquad (5)$$

$$\langle n|j^{(1)}|n\rangle \;\rightarrow\; -G_E^{1}(Q^2)+G_E^{\not{1}}(Q^2)\ , \qquad (6)$$

where the arrow indicates that only leading contributions are listed in the non-relativistic limit of these matrix elements. While higher order corrections associated with the Darwin-Foldy and spin-orbit terms are not displayed explicitly in the equations above, they are in fact retained in the calculations discussed later in the present work. The form factors $G_E^{\not{0}}(Q^2)$ and $G_E^{\not{1}}(Q^2)$ parameterize ISB effects in the nucleon states. We also introduce the strange form factor via

$$\langle p|j^{(s)}|p\rangle=\langle n|j^{(s)}|n\rangle \rightarrow G_E^{s}(Q^2)\ , \qquad (7)$$

where here ISB terms in the p,n states are neglected. Contributions from sea quarks heavier than strange are also ignored.

In terms of the experimental proton and neutron electric form factors, derived from the matrix elements $\langle p|j^{\mu=0}_{\rm EM}|p\rangle \rightarrow G_E^p(Q^2)$ and $\langle n|j^{\mu=0}_{\rm EM}|n\rangle \rightarrow G_E^n(Q^2)$, we obtain:

$$G_E^0 \;=\; (G_E^p+G_E^n)/2-G_E^{\not{1}}\ , \qquad (8)$$

$$G_E^1 \;=\; (G_E^p-G_E^n)/2-G_E^{\not{0}}\ , \qquad (9)$$

where the Q^2 dependence in these and the following two equations is understood. In the limit in which the p,n states form an isospin doublet, the form factors $G_E^{\not{0}}$ and $G_E^{\not{1}}$ vanish, and G_E^0 and G_E^1 reduce to the standard isoscalar and isovector combinations of the proton and neutron electric form factors. The proton and neutron vector NC form factors follow from Eq. (2), *i.e.*

$$G_E^{p,Z}=(1-4s_W^2)G_E^p-G_E^n+2(G_E^{\not{1}}-G_E^{\not{0}})-G_E^s\ , (10)$$

$$G_E^{n,Z}=(1-4s_W^2)G_E^n-G_E^p+2(G_E^{\not{1}}+G_E^{\not{0}})-G_E^s\ . (11)$$

We now turn to the nuclear charge operator. At low momentum transfer, it is simply given by

$$\rho^{(\rm EM)}({\bf q})=G_E^p(Q^2)\sum_{k=1}^{Z}e^{i{\bf q}\cdot{\bf r}_k}+G_E^n(Q^2)\sum_{k=Z+1}^{A}e^{i{\bf q}\cdot{\bf r}_k}\ , \qquad (12)$$

where Z is the number of protons, $A-Z$ the number of neutrons, and for elastic scattering from a nuclear target of mass m_A the squared four-momentum transfer is taken as $Q^2=2\,m_A(\sqrt{q^2+m_A^2}-m_A)$, with ${\bf q}$ being the three-momentum transfer, and $q=|{\bf q}|$. An equation similar to Eq. (12) holds for the weak vector charge operator, but with G_E^p and G_E^n being replaced respectively by $G_E^{p,Z}$ and $G_E^{n,Z}$. It is also convenient to define the charge operators:

$$\rho^{(0)}({\bf q}) \;=\; \frac{G_E^p+G_E^n}{2}\sum_{k=1}^{A}e^{i{\bf q}\cdot{\bf r}_k}\ , \qquad (13)$$

$$\rho^{(1)}({\bf q}) \;=\; \frac{G_E^p-G_E^n}{2}\left(\sum_{k=1}^{Z}e^{i{\bf q}\cdot{\bf r}_k}-\sum_{k=Z+1}^{A}e^{i{\bf q}\cdot{\bf r}_k}\right)\ , \qquad (14)$$

from which

$$\rho^{(\rm EM)}({\bf q}) \;=\; \rho^{(0)}({\bf q})+\rho^{(1)}({\bf q})\ , \qquad (15)$$

$$\rho^{(\rm NC)}({\bf q}) \;=\; -4s_W^2\rho^{(\rm EM)}({\bf q})+\frac{2\,G_E^{\not{1}}-G_E^s}{(G_E^p+G_E^n)/2}\rho^{(0)}({\bf q}) +2\rho^{(1)}({\bf q})-\frac{2\,G^{\not{0}}}{(G_E^p-G_E^n)/2}\rho^{(1)}({\bf q})\ , \qquad (16)$$

where again the Q^2 dependence of the nucleon form factors has been suppressed here and in the following for brevity. The relations above lead to the definition of the following nuclear form factors:

$$\langle ^4{\rm He}|\rho^{(a)}({\bf q})|^4{\rm He}\rangle/Z\equiv F^{(a)}(q)\ ,\quad a={\rm EM},0,1\ , \qquad (17)$$

having the normalizations $F^{(\rm EM)}(0)$=$F^{(0)}(0)$=1 and $F^{(1)}(0)$=0. The form factor $F^{(1)}(q)$ is very small because ^{4}He is predominantly an isoscalar state. Thus, ignoring second order terms like $G^{\not{0}}\,F^{(1)}(q)$, we obtain for the PV asymmetry measured in $(\vec{e},e')$ elastic scattering from ^{4}He:

$$A_{PV}=\frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}}\left[4s_W^2-2\,\frac{F^{(1)}(q)}{F^{(0)}(q)}-\frac{2\,G_E^{\not{1}}-G_E^s}{(G_E^p+G_E^n)/2}\right]\ , \qquad (18)$$

where G_μ is the Fermi constant as determined from muon decays, and here s_W^2 is taken to incorporate radiative corrections. The terms $G_E^{\not{1}}$ and $F^{(1)}(q)/F^{(0)}(q)$ are the contributions to A_{PV}, associated with the violation of isospin symmetry at the nucleon and nuclear level, respectively.

The most accurate measurement of the PV asymmetry, recently reported in Ref. [6] at Q^2=0.077 (GeV/c)^2, gives A_{PV} = [+6.40 ± 0.23 (stat) ± 0.12 (syst)] ppm, from which, after inserting the values for G_μ=1.16637 × 10^{-5} GeV^{-2}, α=1/137.036, and s_W^2=0.2286 (including its radiative corrections [7]) in Eq. (18), one obtains

$$\Gamma\equiv -2\frac{F^{(1)}(q)}{F^{(0)}(q)}-\frac{2\,G_E^{\not{1}}-G_E^s}{(G_E^p+G_E^n)/2}=0.010\pm 0.038 \qquad (19)$$

at Q^2 = 0.077 (GeV/c)^2. This result is consistent with zero. In the following, we discuss the estimates for the ISB corrections first in the nucleon and then in ^{4}He, respectively $G_E^{\not{1}}(Q^2)$ and $F^{(1)}(q)$, at Q^2=0.077 (GeV/c)^2 (corresponding to q=1.4 fm^{-1}).

For $G_E^{\not{1}}(Q^2)$ we use the estimate obtained in Ref. [9] adapted to our conventions, combining a leading-order calculation in chiral perturbation theory with estimates for low-energy constants using resonance saturation. Collecting the various pieces, we find

$$G_E^{\not{1}}(Q^2)=-\frac{g_A^2 m_N\Delta m}{F_\pi^2}\left\{\frac{M_\pi}{m_N}\left(\overline{\gamma}_0(-Q^2)-4\overline{\gamma}_3(-Q^2)\right)\right.$$

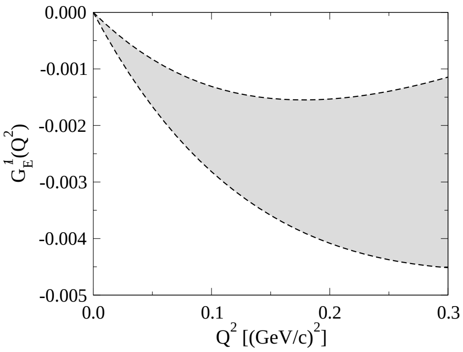

FIG. 1: The isospin-violating nucleon form factor $G_E^{\not{I}}(Q^2)$. The band comprises a range of values for various vector-meson coupling constants, as well as an estimate of higher-order chiral corrections. For details, see Ref. [9].

$$- \frac{Q^2}{2m_N^2}\left[\xi(-Q^2) - \frac{M_\pi}{m_N}\left(\overline{\gamma}_0(-Q^2) - 5\overline{\gamma}_3(-Q^2)\right) - \frac{1}{16\pi^2}\left(1 + 2\log\frac{M_\pi}{M_V} - \frac{\pi(\kappa^v+6)M_\pi}{2m_N}\right)\right]\Bigg\} + \frac{g_\omega F_\rho \Theta_{\rho\omega} Q^2}{2M_V(M_V^2+Q^2)^2}\left(1 + \frac{\kappa_\omega M_V^2}{4m_N^2}\right) , \quad (20)$$

where the loop functions ξ, $\overline{\gamma}_{0/3}$ are given explicitly in Ref. [9], along with the precise definitions of the various coupling constants. The chiral loop contributions in Eq. (20) scale with the neutron–proton mass difference Δm, while the resonance part is proportional to the ρ–ω mixing angle $\Theta_{\rho\omega}$. We refer to Ref. [9] for a detailed discussion of the range of numerical values for the vector meson coupling constants and only show the resulting band for $G_E^{\not{I}}(Q^2)$ in Fig. 1. At the specific kinematical point of interest Q^2=0.077 (GeV/c)^2, we find $G_E^{\not{I}}(Q^2) = -0.0017 \pm 0.0006$, and with $G_E^p(Q^2)$=0.799 and $G_E^n(Q^2)$=0.027 [10], we obtain

$$- \frac{2\,G_E^{\not{I}}}{(G_E^p + G_E^n)/2} = 0.008 \pm 0.003 \quad (21)$$

at $Q^2 = 0.077$ (GeV/c)^2.

We now turn to the nuclear ISB corrections. An approximate calculation of the ratio $F^{(1)}(q)/F^{(0)}(q)$ was carried out more than a decade ago [11], by i) taking into account only the isospin admixtures induced by the Coulomb interaction, ii) constructing a T=1 J^π=0$^+$ breathing mode excitation based on a plausible ansatz, and iii) generating the relevant T=1 component in the ^{4}He ground state in first order perturbation theory. The calculated value was found to be rather small, and it produced a less than 1% correction with respect to the $4\,s_W^2$ term in Eq. (18) at low Q^2.

Since that pioneering study, significant progress has occurred on several fronts. First, there now exist a number of accurate models of nucleon-nucleon (NN) potentials [12, 13, 14, 15, 16] which include explicit ISB induced by both the strong and electromagnetic interactions. These ISB terms have been constrained by fitting pp and np elastic scattering data. It is now an established fact that a realistic study of ^{4}He, and in fact light nuclei [17], requires the inclusion of three-nucleon (NNN) potentials in the Hamiltonian. While these are still not well known, the models most commonly used in the literature [17, 18, 19, 20] do not contain ISB terms. The strength of the latter, however, is expected to be tiny.

Second, several accurate methods have been developed to compute ^{4}He wave functions starting from a given realistic nuclear Hamiltonian [21]. In these calculations, $T > 0$ components are generated non-perturbatively. The T=1 percentage in the ^{4}He wave function is typically found to be of the order of 0.001 %.

In this paper, we use the Hyperspherical Harmonic (HH) expansion method to compute the ^{4}He wave function [22, 23, 24]. In order to have an estimate of the model dependence, we consider a variety of Hamiltonian models, including: i) the Argonne v_{18} NN potential [13] (AV18); ii) the AV18 plus Urbana-IX NNN potential [18] (AV18/UIX); iii) the CD Bonn [14] NN plus Urbana-IXb NNN potentials (CDB/UIXb); and iv) the chiral N3LO [15] NN potential (N3LO). The Urbana UIXb NNN potential is a slightly modified version of the Urbana UIX (in the UIXb, the parameter U_0 of the central repulsive term has been reduced by the factor 0.812), designed to reproduce, when used in combination with the CD Bonn potential, the experimental binding energy of ^{3}H. The binding energies B and $P_{T=1}$ percent probabilities obtained with the AV18, AV18/UIX, CDB/UIXb, and N3LO are respectively B=(24.21, 28.47, 28.30, 25.38) MeV (to be compared with an experimental value of 28.30 MeV) and $P_{T=1}$=(0.0028, 0.0025, 0.0020, 0.0035). These results are in agreement with those obtained with other methods (for a comparison, see Ref. [23]).

The form factors $F^{(0)}(q)$ and $F^{(1)}(q)$, defined in Eq. (17) and calculated with the AV18/UIX Hamiltonian model, are displayed in Fig. 2. The dashed (solid) curves represent the results of calculations including the one-body (one-body plus MEC) EM charge operators (note that ISB corrections in the nucleon form factors entering the two-body EM charge operators, listed explicitly in Ref. [8], are neglected). Similar results (not shown in Fig. 2 to reduce clutter) are obtained with the other Hamiltonian models. In particular, the model dependence in the calculated $F^{(0)}(q)$ form factor is found to be weak, although the change of sign in the predictions corresponding to the N3LO model occurs at a slightly lower value of momentum transfer than in those corresponding to the other models, which are in excellent agreement with the experimental data from Refs. [25]. From the figure it is evident that for $q \leq 1.5$ fm^{-1}, the effect of MEC in both $F^{(0)}(q)$ and $F^{(1)}(q)$ is negligible.

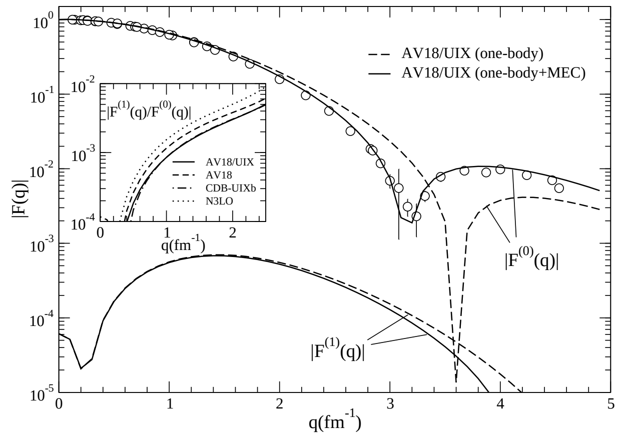

FIG. 2: The $F^{(0)}(q)$ and $F^{(1)}(q)$ form factors for the AV18/UIX Hamiltonian model. The $F^{(0)}(q)$ is compared with the experimental ^{4}He charge form factor [25]. The ratio $|F^{(1)}(q)/F^{(0)}(q)|$ (all calculations include MEC) is shown in the inset for the four Hamiltonian models considered in this paper.

In the inset of Fig. 2, we show the model dependence of the ratio $|F^{(1)}(q)/F^{(0)}(q)|$ (all calculations include MEC). The various Hamiltonian models give predictions quite close to each other, although the value for the N3LO is somewhat larger than for the other models, reflecting the larger percentage of T=1 admixtures in the ^{4}He ground state, predicted by the N3LO potential. The calculated ratios $F^{(1)}(q)/F^{(0)}(q)$ at Q^2=0.077 (GeV/c)^2 are of the order of -0.002. The inclusion of NNN potentials tends to reduce the magnitude of $F^{(1)}/F^{(0)}$, while ignoring MEC contributions, at this value of Q^2, would lead, at the most, to 1.5% decrease of this magnitude.

Note that the value estimated in Ref. [11] was $|F^{(1)}/F^{(0)}| \approx 0.0014$ at Q^2=0.077 (GeV/c)^2, although it was computed in first order perturbation theory by only keeping the ISB corrections due to the Coulomb potential. However, the latter only account for roughly 50 % of the $P_{T=1}$ probability in the ^{4}He ground state [23], and, assuming the ratio above to scale with $\sqrt{P_{T=1}}$, one would have expected a smaller value for it than actually obtained (≈ 0.0014) in Ref. [11].

Therefore, at Q^2=0.077 (GeV/c)^2, both contributions $F^{(1)}/F^{(0)}$ and $G_E^{\not{I}}$ are found of the same order of magnitude as the central value of Γ in Eq. (19). Using in this equation the value $F^{(1)}/F^{(0)} \approx -0.00157$ obtained with the Hamiltonian models including NNN potentials, and the chiral result for $G_E^{\not{I}} = -0.0017 \pm 0.0006$, one would obtain $G_E^s\left[Q^2 = 0.077\,(\mathrm{GeV/c})^2\right] = -0.001 \pm 0.016$ thus suggesting that the value of Γ is almost entirely due to isospin admixtures. Of course, the experimental error on Γ is still too large to allow us to draw a more definite conclusion. A recent estimate of G_E^s using lattice QCD input obtains [26] $G_E^s[0.1\ (\mathrm{GeV/c})^2] = +0.001 \pm 0.004 \pm 0.003$. An increase of one order of magnitude in the experimental accuracy would be necessary in order to be sensitive to G_E^s at low values of Q^2. Indeed, if the lattice QCD prediction above is confirmed, the present data would suggest that the leading correction to the PV asymmetry is from isospin admixtures in the nucleon and/or ^{4}He.

The work of R.S. is supported by the U.S. Department of Energy under contract DE-AC05-06OR23177. B.K. acknowledges partial financial support by the EU I3HP Project (RII3-CT-2004-506078) and DFG (SFB/TR 16), while R.L. is supported in part by the Natural Sciences and Engineering Research Council of Canada, and the Canada Research Chairs Program.

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